

*Benchmark Critical Experiments of  
Uranium-233 Spheres Surrounded  
by Uranium-235*

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**BENCHMARK CRITICAL EXPERIMENTS OF  
URANIUM-233 SPHERES SURROUNDED BY URANIUM-235**

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## **BENCHMARK CRITICAL EXPERIMENTS OF URANIUM-233 SPHERES SURROUNDED BY URANIUM-235**

**IDENTIFICATION NUMBER:** U233-MET-FAST-002

**KEY WORDS:** acceptable, critical experiment, highly enriched  $^{235}\text{U}$  surroundings, metal, sphere,  $^{233}\text{U}$ ,  $^{235}\text{U}$  reflector,  $^{233}\text{U}$  sphere,

### **1.0 DETAILED DESCRIPTION**

#### **1.1 Overview of Experiment**

Evaluated in this report are two critical experiments. Both critical experiments were performed at Los Alamos Scientific Laboratory using spherical masses of uranium highly enriched in  $^{233}\text{U}$  surrounded by uranium highly enriched in  $^{235}\text{U}$ . The first experiment was performed using a core mass of 10.012 kg and will be referred to as the 10 kg experiment. The second experiment was performed using a core mass of 7.601 kg and will be referred to as the 7.6 kg experiment. These experiments are considered to be acceptable as benchmark critical experiments. This experiment was performed as part of a series of experiments. This series is covered by PU-MET-FAST-005, PU-MET-FAST-010, PU-MET-FAST-018, U233-MET-FAST-003, U233-MET-FAST-004, U233-MET-FAST-005, and MIX-MET-FAST-001

#### **1.2 Description of Experimental Configuration**

Both the 7.6 kg and 10 kg experiments were performed in 1958 using the Planet universal assembly machine (Reference 1). The core was composed of two hemispheres with a 0.85-inch-diameter central source cavity. The hemispheres were constructed of uranium highly enriched in  $^{233}\text{U}$  and plated with 0.005 inches of nickel. Hemishells of various thicknesses of uranium highly enriched in  $^{235}\text{U}$  were constructed to enclose the  $^{233}\text{U}$  hemispheres (Reference 1).

A drawing of the Planet assembly machine is shown in Figure 1, and the core setup is shown in Figure 2. The top half of the core rested on a 0.015-inch-thick stainless-steel diaphragm. The lower half of the core rode on a hydraulic lift. The lift that supported the lower half of the assembly was a hollow cylinder made of aluminum. Assembly was accomplished by raising the hydraulic lift which supported the lower half of the core. Rapid disassembly was accomplished by dropping the lift that supported the lower half of the core (Reference 1).



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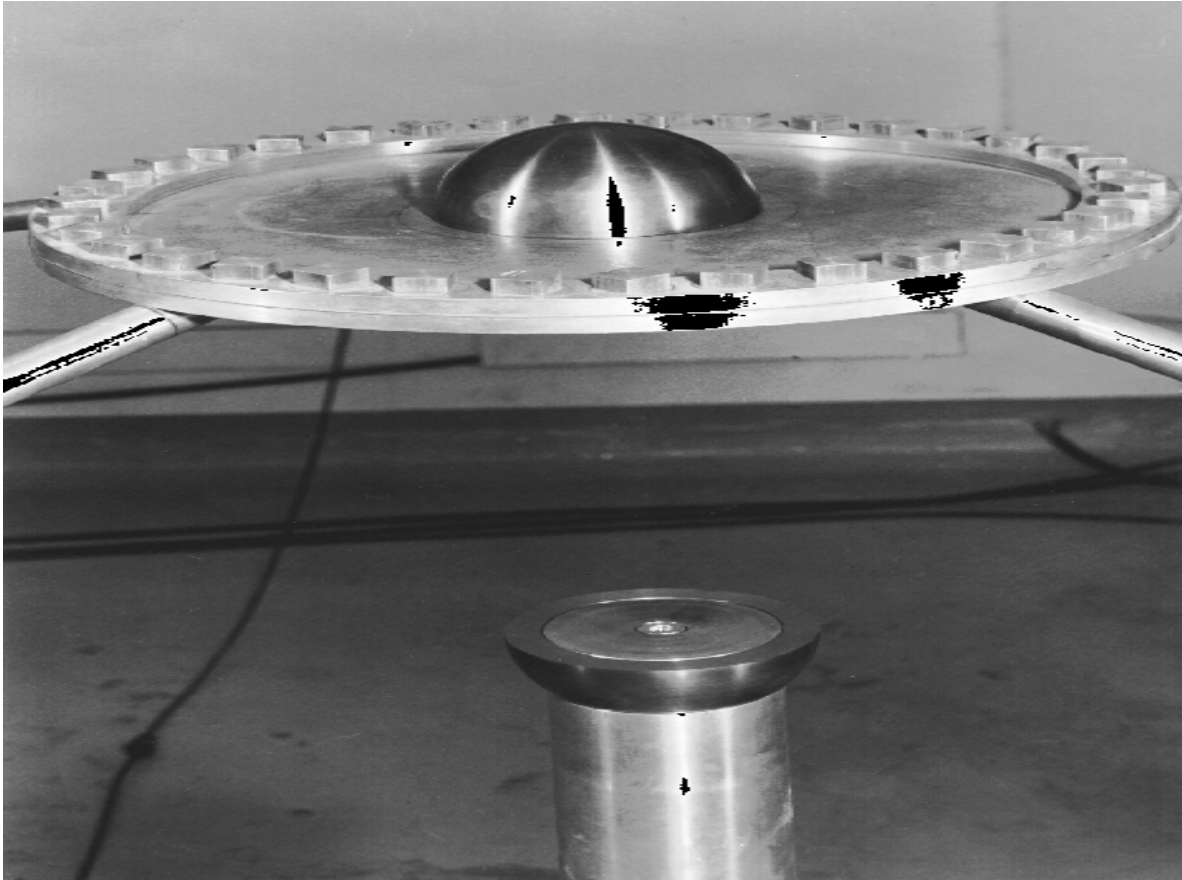


Figure 2. The Core Setup.

Four BF<sub>3</sub> detectors encased in polyethylene were mounted on the assembly lift in such a way as to adequately monitor neutron leakage. A <sup>210</sup>PoBe neutron source with a strength of approximately 10<sup>5</sup> neutrons/sec was placed inside of the source cavity in the upper half of the core. The experiments were performed on the same day such that the short half-life of the source (138 days) was not a factor.<sup>a</sup>

Hemishells were fabricated of uranium highly enriched in <sup>235</sup>U to enclose the <sup>233</sup>U hemispheres, and the multiplication<sup>b</sup> of the assembled system was measured. The procedure was followed for (1) the bare core, (2) an intermediate thickness of <sup>235</sup>U, and (3) a final <sup>235</sup>U thickness. For the 7.6 kg experiment, measurements were made with a 42.52 g close-fitting hemispherical <sup>233</sup>U filler piece in the lower source cavity (Reference 2). For the 10 kg experiment, measurements were made with a 29.64 g close-fitting hemispherical plutonium filler piece in the lower source cavity (Reference 3).

Table 1 shows the actual experimental results for the 10 kg and 7.6 kg experiments. The multiplication measurements were obtained with the filler piece in place, and both halves of the core in direct contact with the diaphragm (References 2 and 3).

Table 1. Multiplication Measurements.

<sup>235</sup> U Thickness (inches)	Core Surroundings Clearance (inches)	Multiplication with filler piece
10 kg Experiment		
0.0	-	6.31
0.383	0.011	24.56
0.490	0.011	84.80
7.6 kg Experiment		
0.0	-	4.560
0.676	0.002	27.69
0.778	0.002	81.89

<sup>a</sup> The PoBe source neutron spectrum and strength changes over time because of radioactive decay, chemical and physical changes, such as recrystallization of the beryllium. If measurements are taken with the same source over a relatively long period of time, the measured multiplication and, therefore, the critical characteristic dimension determination can be affected.

<sup>b</sup> Multiplication, in simple terms, is the ratio of the neutron count rates as measured by external neutron counters, with and without fissile material present. That is, in a subcritical system with a neutron source, multiplication is the equilibrium ratio of the total number of fission and source neutrons to the total number of source neutrons.



Additional experimental measurements and corrections were made to the experimental configuration to obtain the critical mass of a solid sphere of uranium highly enriched in  $^{233}\text{U}$  with close fitting spherical  $^{235}\text{U}$  surroundings. These corrections and measurements are described in Section 2.

### 1.3 Description of Material Data

The core composition is given in Table 2. The isotopic compositions are the same for both cores (Reference 5). The density of the 7.6 kg core was  $18.644 \text{ g/cm}^3$  and  $18.621 \text{ g/cm}^3$  for the 10 kg core.

Table 2. Core Composition.

Isotope	Wt. %
$^{233}\text{U}$	98.2
$^{234}\text{U}$	1.1
$^{238}\text{U}$	0.7

The average composition of the surrounding  $^{235}\text{U}$  is given in Table 3 (Reference 5). The  $^{235}\text{U}$  composition is the same composition for the two experiments, so only one set of compositions is given in Table 3. The density of the surrounding  $^{235}\text{U}$  is  $18.80 \text{ g/cm}^3$  for both experiments.

Table 3. Composition of the Highly Enriched  $\text{U}^{235}$  Surroundings.

Isotope	Wt. %
$^{235}\text{U}$	93.2
$^{238}\text{U}$	6.8

Trace impurities in these materials were not given but were stated to be similar to those found in the Godiva and the Jezebel assemblies and, as such, have a negligibly small effect on the critical specifications (Reference 5).

#### **1.4 Supplemental Experimental Measurements**

No additional experimental measurements were completed with the exception of those discussed in Section 2.

## 2.0 EVALUATION OF EXPERIMENTAL DATA

The need and value of solid spherical critical assemblies was foreseen by the experimenters, and, thus, the experimenters made some additional corrections and measurements to obtain solid spherical core critical masses.

First, the experimenters determined the effect of the 0.015-inch-thick diaphragm. The results of the study indicated that the steel diaphragm had little effect other than maintaining a gap between the two halves of the assembly. The measured  $\Delta 1/M$  effect of the diaphragm was approximately -0.0036.

"A plot of reciprocal multiplication ( $1/M$ ) was extrapolated for an additional 0.015 inches past the closure point to obtain the effect of the diaphragm on the multiplication of the system. The validity of this extrapolation was verified by increasing the thickness of the diaphragm with an additional 0.015-inch sheet of stainless steel. The multiplication thus obtained agreed with what could have been predicted from the  $1/M$  closure curve . . ." (Reference 1).

Figure 3 was reproduced from Reference 1 and shows the plot of reciprocal multiplication versus  $^{235}\text{U}$  surrounding thickness.

Next, the change in multiplication was measured when a close-fitting filler piece was added to the bottom half of the central source cavity. From this measurement, a good estimate was

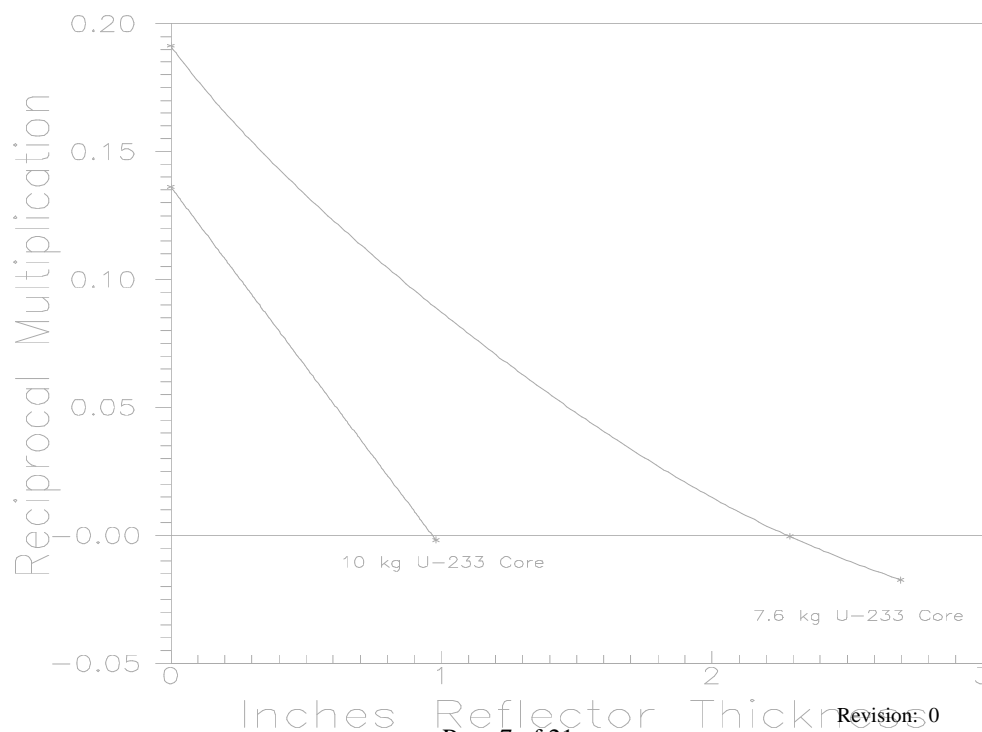


Figure 3. Corrected Reciprocal Multiplications for the  $^{233}\text{U}$  Cores Surrounded by  $^{235}\text{U}$ .

made of the expected change in multiplication when the source cavity was completely filled (i.e., solid spherical critical masses) (References 6 and 7). The  $\Delta 1/M$  correction for the central void was -0.0100. This value includes the correction for the nickel on the source cavity surface.

The effect of the 0.005-inch-thick nickel plating on the hemisphere parting plane and the source cavity surface was determined using material replacement data made on the  $^{239}\text{Pu}$  Jezebel bare critical assembly. The worth of the nickel on the interior of the surface cavity was previously mentioned, and the  $\Delta 1/M$  for the nickel on the parting plane was determined to be -0.0040. The effect of the nickel on the surface of the  $^{233}\text{U}$  core was estimated using data from the Topsy critical assembly for which the 0.005-inch-thick nickel plating is equivalent to a 0.0025-inch-thick layer of uranium (Reference 1).

The magnitudes of the previously described corrections made for the diaphragm, source cavity, and internal nickel are applicable only to the 10 kg experiment. The magnitudes of these corrections for the 7.6 kg experiment were not given in the references.

An approximate correction was made for clearances between the core and the surroundings as described below. Explicit values for this correction in terms of  $\Delta 1/M$  were not given in the references.

"A suitable correction for the clearance between the core and reflector material, which was present in the experimental setup, was deduced from the slope of the measured curve of reciprocal multiplication versus reflector thickness. The slope at zero thickness is interpreted as giving the change in  $1/M$  per unit thickness which would have occurred had the clearance void been filled with reflector. The external reflector radius was reduced to produce this same  $\Delta(1/M)$  as indicated by the slope of the curve in this region." (Reference 1)

Care was taken by the experimenters to ensure that the equatorial surfaces of the lower core hemisphere and the lower  $^{235}\text{U}$  hemishell were flush by placing small pieces of shim stock between the polar surfaces of the lower core and  $^{235}\text{U}$  parts (Reference 8). This was done to ensure that both pieces (core and  $^{235}\text{U}$ ) were in direct contact with the diaphragm.

With the above measurements and corrections, the experimenters were able to correct their inverse multiplication versus  $^{235}\text{U}$  thickness curve to that for solid spherical cores of  $^{233}\text{U}$  surrounded by close-fitting  $^{235}\text{U}$  surroundings. The resulting critical uranium thicknesses were found to be 0.481 inches and 0.783 inches for the two cores (Reference 5). These thicknesses were found by interpolating between the data shown in Figure 3.

Later efforts made corrections to account for reflection by the Planet assembly machine and the surrounding room walls. This correction increased the HEU thickness by 0.003 inches (Reference 3).

It was observed that the experimenters used many corrections particular to  $^{239}\text{Pu}$  metal ( $^{239}\text{Pu}$  Jezebel and Topsy) for the  $^{233}\text{U}$  metal systems. The experimenters also used a plutonium filler piece when measurements were taken to reduce the 10 kg experiment to a solid sphere.

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This is the reason why the multiplications reported in Reference 3 differ from those in Reference 6. Previous experiments indicated that  $^{239}\text{Pu}$  and  $^{233}\text{U}$  critical assemblies are similar (Reference 9). Another indication of the validity of this substitution can be seen in the similar critical masses of the two isotopes: 17.02 kg for  $^{239}\text{Pu}$  metal (PU-MET-FAST-001) and 16.53 kg for  $^{233}\text{U}$  metal (U233-MET-FAST-001). A small correction was applied to correct for using plutonium instead of  $^{233}\text{U}$ .

The reported core densities, core masses, and core enrichments are different for the three primary references (References 1, 4, and 5). It was deduced from References 1, 4, and 5 that the same cores were used in the experiments performed to determine critical thickness for several reflector materials. The reported data are summarized in Table 4. The reported critical masses in Reference 4 are believed to be typographical errors because the masses correspond to the uncorrected core masses. Furthermore, the masses are not consistent with the reported core diameter and the reported core densities in that reference, and the core diameters and densities in Reference 4 correspond to the masses reported in References 1 and 5. The discrepancies between core enrichments were studied, and the enrichments reported in Reference 5 are used for the benchmark model. Besides the fact that Reference 5 is the most recent reevaluation of these experiments, it also contains the most complete and most consistent data, as shown in Table 4.

Table 4. Differences Between the Reported Specifications of the Three Primary References.

Parameter	Reference 1	Reference 5	Reference 4
$^{233}\text{U}$ wt. %	98.25	98.2	98.2
$^{234}\text{U}$ wt. %	none given	1.1	1.1
$^{235}\text{U}$ wt. %	none given	none given	none given
$^{238}\text{U}$ wt. %	none given	0.7	0.7
10 kg Density ( $\text{g}/\text{cm}^3$ )	18.62	18.621	18.62
10 kg Mass (g)	10012	10012	9840
7.6 kg Density ( $\text{g}/\text{cm}^3$ )	18.62	18.644	18.64
7.6 kg Mass (g)	7601	7601	7470

The 10 kg idealized critical experiment, as corrected, consists of a 3.972-inch-diameter  $^{233}\text{U}$  spherical core (density =  $18.621 \text{ g}/\text{cm}^3$ ) intimately surrounded by a spherical shell of  $^{235}\text{U}$ , 0.481-inches thick, with a density of  $18.80 \text{ g}/\text{cm}^3$ . The 7.6 kg idealized critical experiment, as corrected, consists of a 3.622-inch-diameter  $^{233}\text{U}$  spherical core (density =  $18.644 \text{ g}/\text{cm}^3$ ) intimately surrounded by a spherical shell of  $^{235}\text{U}$ , 0.783-inches thick, with a density of  $18.80 \text{ g}/\text{cm}^3$ . The core specifications as described in the reevaluation efforts of Reference 5 are

what is described in this report. It is these descriptions that we accept as the benchmark models. The sensitivity of the calculational benchmark models to various parameters is studied in Appendix B.

### 3.0 BENCHMARK SPECIFICATIONS

#### 3.1 Description of Model

**3.1.1 The 10 kg Experiment** - The benchmark model is a simple highly enriched  $^{233}\text{U}$  sphere with a density of  $18.621 \text{ g/cm}^3$  and a core mass of 10012 grams uranium surrounded by  $1.2217 \pm 1\%$  cm highly enriched  $^{235}\text{U}$  at a density of  $18.80 \text{ g/cm}^3$  (Reference 5). The model is an idealized configuration derived by the experimenters.

**3.1.2 The 7.6 kg Experiment** - The benchmark model is a simple highly enriched  $^{233}\text{U}$  sphere with a density of  $18.644 \text{ g/cm}^3$  and a core mass of 7601 grams uranium surrounded by  $1.9888 \pm 1\%$  cm highly enriched  $^{235}\text{U}$  at a density of  $18.80 \text{ g/cm}^3$  (Reference 5). The model is an idealized configuration derived by the experimenters.

#### 3.2 Dimensions

**3.2.1 The 10 kg Experiment** - The radius of the  $^{233}\text{U}$  core, a 10012 gram uranium sphere at a density of  $18.621 \text{ g/cm}^3$ , was 5.0444 cm. The sphere was surrounded by 1.2217 cm of highly enriched uranium (outer radius of the surroundings is 6.2661 cm).

**3.2.2 The 7.6 kg Experiment** - The radius of the  $^{233}\text{U}$  core, a 7601 gram uranium sphere at a density of  $18.644 \text{ g/cm}^3$ , was 4.5999 cm. The sphere was surrounded by 1.9888 cm of highly enriched uranium (outer radius of the surroundings is 6.5887 cm).

#### 3.3 Material Data

The calculated atomic number densities of the uranium cores for the isotopic compositions given previously in Tables 2 and 3 are shown in Table 5 using the densities given in Section 3.2.

Table 5. Atom Densities for the <sup>233</sup>U Spheres.

Isotope	Atom Density, 10 kg (atoms/barn-cm)	Atom Density, 7.6 kg (atoms/barn-cm)
<sup>233</sup> U Core		
<sup>233</sup> U	$4.7253 \times 10^{-2}$	$4.7312 \times 10^{-2}$
<sup>234</sup> U	$5.2705 \times 10^{-4}$	$5.2770 \times 10^{-4}$
<sup>238</sup> U	$3.2975 \times 10^{-4}$	$3.3015 \times 10^{-4}$
<sup>235</sup> U Surroundings		
<sup>235</sup> U	$4.4892 \times 10^{-2}$	$4.4892 \times 10^{-2}$
<sup>238</sup> U	$3.2340 \times 10^{-3}$	$3.2340 \times 10^{-3}$

### 3.4 Temperature Data

No mention was made in the references in regard to experimental temperature. The experimental temperature was assumed to be room temperature (293 kelvin).

### 3.5 Experimental and Benchmark-Model $k_{\text{eff}}$

**3.5.1 The 10 kg Experiment** - At a thickness of 0.490 inches, the measured multiplication for this experiment was 84.80 and the corrected inverse multiplication -0.0052 (Reference 3). The difference between quoted (-0.0052) and inferred (-0.0058) inverse multiplications is 0.0006, which is less than the experimental uncertainty,  $\pm 0.0010$  (see Appendix B). Corrections were also made for external nickel and for the clearance between the core and reflector. The correction for the clearances was -0.002 inches from that in Reference 3. The correction for external nickel was +0.0025 inches. Both of these effects would combine for a net positive effect, which would further decrease the difference between quoted and inferred inverse multiplications. The actual net worth of these two corrections in terms of  $\Delta 1/M$  was not given in the references.

The benchmark model  $k_{\text{eff}}$  is  $1.0000 \pm 0.0010$ , where the uncertainty in  $k_{\text{eff}}$  is due to the  $\pm 1.0\%$  uncertainty in the thickness of the surroundings. The uncertainty in  $k_{\text{eff}}$  is derived from a ONEDANT calculation shown in Appendix B.

**3.5.2 The 7.6 kg Model** - At a thickness of 0.778 inches, the measured multiplication for this experiment was 81.89 and the corrected inverse multiplication -0.0010 (Reference 2). Differences between quoted and inferred inverse multiplications are not shown here because the magnitude of corrections for the 7.6 kg experiment was not given in the references. The



experimental  $k_{\text{eff}}$  is 1.0000, which includes experimental corrections made for the diaphragm, the source cavity, and internal and external nickel.<sup>a</sup> The benchmark-model  $k_{\text{eff}}$  is  $1.0000 \pm 0.0011$ , where the uncertainty in  $k_{\text{eff}}$  corresponds to the  $\pm 1\%$  uncertainty in the thickness of the surroundings. The uncertainty in  $k_{\text{eff}}$  is derived from a ONEDANT calculation shown in Appendix B.

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<sup>a</sup> Inverse multiplication,  $1/M$ , is related to  $k_{\text{eff}}$  by:  $1/M = 1 - k_{\text{eff}}$ . This relationship is valid if  $k_{\text{eff}}$  is "close to one." The statement, "close to one," is meant to approximate a critical system, and it varies from system to system.

#### 4.0 RESULTS OF SAMPLE CALCULATIONS

In this section, results of calculations using various criticality codes and cross section sets are given. The results of these calculations using the models which were described previously in Section 3.2 are shown in Table 6. The calculations using 27-group SCALE cross sections significantly underpredict  $k_{\text{eff}}$ , as seen in Table 6. This prediction is due to known deficiencies in the  $^{233}\text{U}$  cross sections for this library.<sup>a</sup>

Table 6. Sample Calculation Results (United States).

Code (Cross Section Set) Case Number ↓	KENO (Hansen-Roach)	KENO (27-Group ENDF/V-IV)	MCNP (Continuous Energy ENDF/B-V)	ONEDANT (27-Group ENDF/B-IV)
1 (10 kg)	1.0010 ± 0.0013	0.9739 ± 0.0011	0.9969 ± 0.0009	0.9724
2 (7.6 kg)	1.0072 ± 0.0012	0.9798 ± 0.0011	1.0007 ± 0.0009	0.9788

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<sup>a</sup> Personal communication, R. M. Westfall and L. M. Petrie, ORNL, September 1993.

## 5.0 REFERENCES

1. E. A. Plassmann and D. P. Wood, "Critical Reflector Thicknesses for U<sup>233</sup> and Pu<sup>239</sup> Systems," Nucl. Sci. Eng., **8**, pp. 615-620, 1960.
2. E. A. Plassmann and D. P. Wood, N-Division Progress Report for September 21 - October 20, 1959, pp. 7-9, Los Alamos National Laboratory Archives A-86-016, 202-5.
3. E. A. Plassmann, N-Division Progress Report for October 21 - November 20, 1957, pp. 22-26, Los Alamos National Laboratory Archives A-86-016, 202-6.
4. H. C. Paxton, "Los Alamos Critical - Mass Data," LA-3067-MS, December 1975.
5. G. E. Hansen and H. C. Paxton, "Reevaluated Critical Specifications of Some Los Alamos Fast-Neutron Systems," LA-4208, p. 8, September 1969.
6. E. A. Plassmann and D. P. Wood, N-Division Progress Report for July 21 - August 20, 1957, pp. 15-18, Los Alamos National Laboratory Archives A-86-016, 200-12.
7. E. A. Plassmann, D. P. Wood and R. E. White, N-Division Progress Report for March 21 - April 20, 1957, pp. 21-23, Los Alamos National Laboratory Archives A-86-016, 200-8.
8. E. A. Plassmann and D. P. Wood, N-Division Progress Report for April 21 - May 20, 1957, pp. 15-18, Los Alamos National Laboratory Archives A-86-016, 200-9.
9. H. C. Paxton, "Critical Masses of Fissionable Metals as Basic Nuclear Safety Data," LA-1958, January 1955.

## APPENDIX A: TYPICAL INPUT LISTINGS

### A.1 KENO Input Listings

#### Hansen-Roach Cross Sections and 27-Group ENDF/B-IV Cross Sections

Listed below are the input listings for KENO V.a with 16-group Hansen-Roach and 27-group ENDF/B-IV SCALE4 cross sections for both experiments. The models use 300 active generations with 1500 histories per generation while skipping the first five generations.

In the parameter section of the input file, "lib=40" is specified. This is the Los Alamos version in AMPX format of the 16-group Hansen-Roach cross-section library.

KENO-V.a Input Listing for Table 6 (16-Energy-Group Hansen-Roach Cross Sections).

```
U-233 SPHERE REFLECTED BY .481 IN U-235
READ PARAMETERS
  RUN=YES FAR=YES LIB=40
  TME=1000 TBA=10
  GEN=305 NPG=1500 NSK=5
END PARAMETERS
READ MIXT SCT=1
MIX=1 92300 .047253
      92400 .00052705
      92800 .00032975
MIX=2 92800 .0032340
      92500 .044892
END MIXT
READ GEOM
UNIT 1
  SPHERE  1 1 5.0444
  SPHERE  2 1 6.2661
END GEOM
END DATA
```

```
U-233 SPHERE REFLECTED BY .783 IN OF U-235
READ PARAMETERS
  RUN=YES FAR=YES LIB=40
  TME=1000 TBA=10
  GEN=305 NPG=1500 NSK=5
END PARAMETERS
READ MIXT SCT=1
MIX=1 92300 .047312
      92400 .00052770
      92800 .00033015
MIX=2 92800 .0032340
      92500 .044892
END MIXT
READ GEOM
UNIT 1
  SPHERE  1 1 4.5999
  SPHERE  2 1 6.5887
END GEOM
END DATA
```

KENO-V.a Input Listing for Table 6 (27-Energy-Group SCALE4 Cross Sections).

```
=CSAS25
U-233 SPHERE WITH .481 IN OF U-235
27GROUPNDF4 INFHOMMEDIUM
U-233  1 0.0 0.047253  END
U-234  1 0.0 0.00052705 END
U-238  1 0.0 0.00032975 END
U-235  2 0.0 0.044892  END
U-238  2 0.0 0.0032340 END
END COMP
U-233 SPHERE
READ PARAMETERS
  TME=1000 TBA=10
  GEN=305 NPG=1500 NSK=5
END PARAMETERS
READ GEOM
UNIT 1
  SPHERE  1 1 5.0444
  SPHERE  2 1 6.2661
END GEOM
END DATA
```

```
=CSAS25
U-233 SPHERE WITH .783 IN U-235
27GROUPNDF4 INFHOMMEDIUM
U-233  1 0.0 0.047312  END
U-234  1 0.0 0.00052770 END
U-238  1 0.0 0.00033015 END
U-235  2 0.0 0.044892  END
U-238  2 0.0 0.0032340 END
END COMP
U-233 SPHERE
READ PARAMETERS
  TME=1000 TBA=10
  GEN=305 NPG=1500 NSK=5
END PARAMETERS
READ GEOM
UNIT 1
  SPHERE  1 1 4.5999
  SPHERE  2 1 6.5887
END GEOM
END DATA
END
```

## A.2 MCNP Input Listings

Listed below are the input files for MCNP 4.2 with continuous-energy ENDF/B-V cross sections for both experiments, with 300 active generations, 1500 histories per generation, and skipping the first 5 generations.

MCNP Input Listing for Table 6.

.481 IN HIGH ENRICHED U-235 REFLECTED U-233 SPHERE

1 1 0.0481098 -1 imp:n=1  
2 2 0.048126 1 -2 imp:n=1  
3 0 2 imp:n=0

1 so 5.0444  
2 so 6.2661

m1 92233.50c 0.047253  
92234.50c 0.00052705  
92238.50c 0.00032975  
m2 92238.50c 0.0032340  
92235.50c 0.044892  
kcode 1500 1.0 5 305  
ksrc 0. 0. 0.  
print

.783IN HIGH ENRICHED U-235 REFLECTED U-233 SPHERE

1 1 0.04816985 -1 imp:n=1  
2 2 0.048126 1 -2 imp:n=1  
3 0 2 imp:n=0

1 so 4.5999  
2 so 6.5887

m1 92233.50c 0.047312  
92234.50c 0.00052770  
92238.50c 0.00033015  
m2 92238.50c 0.0032340  
92235.50c 0.044892  
kcode 1500 1.0 5 305  
ksrc 0. 0. 0.  
print

### A.3 ONEDANT Input Listings

Listed below are the input files for ONEDANT version 2.3e for both experiments. The models use the 27-group SCALE4 ENDF/B-IV cross sections which have  $P_3$  scatter data. The quadrature set is  $S_{48}$ . The convergence criteria is  $10^{-4}$  for eigenvalue and flux by default. The mesh size is approximately 20 mesh/cm in both the  $^{233}\text{U}$  core and the HEU surroundings.

ONEDANT Input Listing for Table 6.

```
=CSASI
ICE RUN TO GET XSECTS FOR U-233 SPHERE 10 KG
27GROUPNDF4 INFHOMMEDIUM
U-233 1 0.0 0.047253 END
U-234 1 0.0 0.00052705 END
U-238 1 0.0 0.00032975 END
U-235 2 0.0 0.044892 END
U-238 2 0.0 0.0032340 END
END COMP
END

2
U233 sphere with a U(93) reflector
Simplified model
/BLOCK 1
igeom=sph ngroup=27 niso=2 isn=-48 mt=2 nzone=2 im=2 it=125 t
/BLOCK 2
xmesh=0.0,5.0444,6.2661 xints=101,24 zones=1,2 t
/BLOCK 3
lib=xs27.w12
maxord=3 ihm=42 iht=3 ihs=16 ititl=1 ifido=2 i2lp1=1 t
/BLOCK 4
matls=isos
assign=matls t
/BLOCK 5
chi=.020 .195 .219 .126 .164 .172 .088 .014 .001 18z;
.021 .188 .215 .125 .166 .180 .090 .014 .001 18z
ievt=1 isct=3 t

=CSASI
ICE RUN TO GET XSECTS FOR U-233 SPHERE .783 IN
27GROUPNDF4 INFHOMMEDIUM
U-233 1 0.0 0.047312 END
U-234 1 0.0 0.00052770 END
U-238 1 0.0 0.00033015 END
U-235 2 0.0 0.044892 END
U-238 2 0.0 0.0032340 END
END COMP
END

2
U233 sphere with a U(93) reflector
Simplified model
/BLOCK 1
igeom=sph ngroup=27 niso=2 isn=-48 mt=2 nzone=2 im=2 it=132 t
/BLOCK 2
xmesh=0.0,4.5999,6.5887 xints=92,40 zones=1,2 t
/BLOCK 3
lib=xs27.w12
maxord=3 ihm=42 iht=3 ihs=16 ititl=1 ifido=2 i2lp1=1 t
/BLOCK 4
matls=isos
assign=matls t
/BLOCK 5
chi=.020 .195 .219 .126 .164 .172 .088 .014 .001 18z;
.021 .188 .215 .125 .166 .180 .090 .014 .001 18z
ievt=1 isct=3 t
```

## APPENDIX B: SENSITIVITY STUDIES

The results of calculations to determine the experimental uncertainties of the calculation models are reported in this Appendix. An experimental uncertainty of  $\pm 1\%$  in the HEU thickness was given by the experimenters. Calculations were performed to determine the  $\Delta k$  associated with this uncertainty for the 10 kg experiment. The result of this calculation is given below in Table B.1 ( $\pm 0.0010$ ).

ONEDANT/TWODANT with Hansen-Roach cross sections was used for the sensitivity studies. The results are shown in Table B.1.

Table B.1. Sensitivity Studies.

Effect	$\Delta k_{\text{eff}}$
10 kg Experiment	
$\pm 1\%$ of $^{235}\text{U}$ thickness <sup>(a)</sup>	$\pm 0.0010$ <sup>(b)</sup>
7.6 kg Experiment	
$\pm 1\%$ of $^{235}\text{U}$ thickness <sup>(a)</sup>	$\pm 0.0011$ <sup>(c)</sup>

- (a) The uncertainty is given as  $\pm 1\%$  in the thickness of the  $^{235}\text{U}$  surroundings.
- (b) The  $\Delta k_{\text{eff}}$  is relative to a base case of 1.0008.
- (c) The  $\Delta k_{\text{eff}}$  is relative to a base case of 1.0022.

The references did not specify a  $^{234}\text{U}$  content for the  $^{235}\text{U}$  surroundings; therefore, calculations were performed to determine the sensitivity of the models to the  $^{234}\text{U}$  content. The results are reported in Table B.2 and B3 for both the 10 kg and the 7.6 kg experiments. ONEDANT was used for the sensitivity studies. As can be seen below, the effect of  $^{234}\text{U}$  content in the applicable range is less than or equal to the  $\Delta k$  associated with experimental uncertainty ( $\pm 0.0010$  and  $\pm 0.0011$ , respectively). Two uranium specifications are analyzed using three different cross section sets. The first uranium specification assumes that the uranium is similar in composition to Godiva (HEU-MET-FAST-001). The second uses the actual uranium specifications given in LA-4208 assuming that the  $^{234}\text{U}$  content is 1.02 wt.% and the  $^{235}\text{U}$  content is 93.2 wt.%.

As can be seen below,  $^{234}\text{U}$  has a negligible effect on the benchmark-model  $k_{\text{eff}}$ . Therefore, the HEU composition as described previously in Section 1.3 is used for the benchmark model.

Table B.2. <sup>234</sup>U Sensitivity Study for the 10 kg Experiment.

Godiva <sup>(a)</sup>	Cross Sections	k <sub>eff</sub>	Δk
0.00 wt.% <sup>234</sup> U	16 Group Hansen-Roach	1.0008	0.0000
0.00 wt.% <sup>234</sup> U	30 Group ENDF/B-V	0.9946	0.0000
1.02 wt.% <sup>234</sup> U	16 Group ENDF/B-IV ( <sup>234</sup> U)	1.0001	-0.0007
1.02 wt.% <sup>234</sup> U	16 Group Mihalcz Modified ( <sup>234</sup> U)	1.0011	+0.0003
1.02 wt.% <sup>234</sup> U	30 Group ENDF/B-V	0.9951	+0.0005
Actual Specifications <sup>(b)</sup>	Cross Sections	k <sub>eff</sub>	Δk
0.00 wt.% <sup>234</sup> U	16 Group Hansen-Roach	0.9998	0.0000
0.00 wt.% <sup>234</sup> U	30 Group ENDF/B-V	0.9912	0.0000
1.02 wt.% <sup>234</sup> U	16 Group ENDF/B-IV ( <sup>234</sup> U)	0.9991	-0.0007
1.02 wt.% <sup>234</sup> U	16 Group Mihalcz Modified ( <sup>234</sup> U)	1.0006	+0.0010
1.02 wt.% <sup>234</sup> U	30 Group ENDF/B-V	0.9922	+0.0010

(a) ρ = 18.74 g/cm<sup>3</sup>, 93.71 wt.% <sup>235</sup>U, 1.02 wt.% <sup>234</sup>U, 5.27 wt.% <sup>238</sup>U.

(b) ρ = 18.80 g/cm<sup>3</sup>, 93.20 wt.% <sup>235</sup>U, 1.02 wt.% <sup>234</sup>U, 5.78 wt.% <sup>238</sup>U.



Table B.3.  $^{234}\text{U}$  Sensitivity Study for the 7.6 kg Experiment.

Godiva <sup>(a)</sup>	Cross Sections	$k_{\text{eff}}$	$\Delta k$
0.00 wt.% $^{234}\text{U}$	16 Group Hansen-Roach	1.0016	0.0000
0.00 wt.% $^{234}\text{U}$	30 Group ENDF/B-V	0.9957	0.0000
1.02 wt.% $^{234}\text{U}$	16 Group ENDF/B-IV ( $^{234}\text{U}$ )	1.0011	-0.0005
1.02 wt.% $^{234}\text{U}$	16 Group Mihalcz Modified ( $^{234}\text{U}$ )	1.0022	+0.0006
1.02 wt.% $^{234}\text{U}$	30 Group ENDF/B-V	0.9959	+0.0002
Actual Specifications <sup>(b)</sup>	Cross Sections	$k_{\text{eff}}$	$\Delta k$
0.00 wt.% $^{234}\text{U}$	16 Group Hansen-Roach	1.0022	0.0000
0.00 wt.% $^{234}\text{U}$	30 Group ENDF/B-V	0.9963	0.0000
1.02 wt.% $^{234}\text{U}$	16 Group ENDF/B-IV ( $^{234}\text{U}$ )	1.0014	-0.0008
1.02 wt.% $^{234}\text{U}$	16 Group Mihalcz Modified ( $^{234}\text{U}$ )	1.0026	+0.0004
1.02 wt.% $^{234}\text{U}$	30 Group ENDF/B-V	0.9968	+0.0005

(a)  $\rho = 18.74 \text{ g/cm}^3$ , 93.71 wt.%  $^{235}\text{U}$ , 1.02 wt.%  $^{234}\text{U}$ , 5.27 wt.%  $^{238}\text{U}$ .

(b)  $\rho = 18.80 \text{ g/cm}^3$ , 93.20 wt.%  $^{235}\text{U}$ , 1.02 wt.%  $^{234}\text{U}$ , 5.78 wt.%  $^{238}\text{U}$ .

